Model Paper

GOVERNMENT COLLEGE UNIVERSITY, FAISALABAD

BA/B.Sc (Part-I) Annual 2017

Subject: Mathematics General Paper: (I) Course Code: MTH-303

Course Code: Calculus (Differntial, Integral, Infinite Series and Calculus)

Time Allowed: 03:00 Hours Maximum Marks: 100 Pass Marks: 33%

Note: Attempt Six questions in all, selecting two questions from section I & II each and One question from III & IV.

SECTION (i)

Q.1 (a) Show that $\lim_{x\to 0}\frac{\cos x-1}{x}=0$ (9,8)

(b)Examine whether the given function is continuous or not at x=0

$$f(x)=\begin{cases}\frac{\sin 3x}{\sin 2x} & if\ x\neq 0\\ \frac{2}{3} & if x=0\end{cases}$$

Q.2 (a) if x>0, prove that : $x-ln(1+x)>\frac{x^2}{2(1+x)}$ (9,8)

(b) if y=$tan^{-1}x$, show that : $(1+x^2)y''+2xy'=0$, Hence find the value of y^n when x=0

Q.3 (a) A boy makes a paper cup in the shape of right circular cone with height four times its radius if radius is changed from 2 cm to 1.5 cm but height remains four times the radius, find the approximate decrease in the capacity of cup. (9,8)

(b) A stone is dropped in a still lake causing circular ripples to spread over the surface. If the radius of the circle increases at the rate of 0.5 meter per second, how fast is the area of ripple increasing when the radius of ripple is 20 meters?

Q.4 (a) Find equation of asymptotes of a curve $r=2a\sin\theta\tan\theta$ (9,8)

(b) Prove that radius of curvature at the point (2a,2a) on the curve $x^2y=a(x^2+y^2)$ is 2a

SECTION(ii)

Q.5 (a) Evaluate $\int x\sin^{-1}x\ dx$ (9,8)

(b) Integrate w.r.t. x $\frac{1}{4\sin x-3\cos x}$

P.T.O

Q.6 (a) Calculate $\int_0^{\pi/2} \ln(\sin x)dx$ (9,8)

(b) Use any numerical integration method to compute $\int_0^2 \frac{1}{\sqrt{1+x^2}}dx$ with n=4.

Q.7 (a) Find area of surface generated by revolving the curve $y = x^3, 0 \le x \le 1/2$ about x-axis (9,8)

(b) Find the area of a region included within cardioid $r = a(1 - \sin\theta)$

Q.8(a) Find the length of a curve $r = \sin^2\left(\frac{\theta}{2}\right)$ from (0,0) to $(1,\pi)$. (9,8)

(b) Sketch the graph $r = 3 + 2\sin\theta$ (limacon)

SECTION (iii)

Q.9 (a) Determine whether the sequence $\sqrt{n}(\sqrt{n+1} - \sqrt{n})$ converges or diverges. (8,8)

(b) Use comparison test to check the series $\sum_1^\infty \frac{1}{n^{1/2}+n^{3/2}}$ converges or diverges.

Q.10 (a) Apply ratio test to determine whether the series $\sum_1^\infty \frac{n}{n^2+1}$ converges or diverges. (8,8)

(b) Obtain the power series representation of $\frac{x}{(1+x^2)^2}$ if $|x| < 1$

SECTION(iv)

Q.11 (a)Find a such that the function $f(x) = \begin{cases} \frac{3xy}{\sqrt{x^2+y^2}} & if\ (x,y) \neq (0,0) \\ a & if\ (x,y) = 0 \end{cases}$ is continuous at (0,0) (8,8)

(b) Let $f(x,y,z) = x^3 + 3yz + \sin xyz$. Prove that $f_{xyz} = f_{zxy}$

Q.12 (a) Use differentials to estimate the value of $\sin 44^0$ and $\tan 44^0$ (8,8)

(b) Find extrema of $f(x,y) = x^2 - e^{y^2}$

Model Paper

GOVERNMENT COLLEGE UNIVERSITY, FAISALABAD

BA/B.Sc (Part-II) Annual 2017

Subject: Mathematics General Paper: (II) Course Code: MTH-403

Course Code: Mathematical Method (Geometry, Complex Number, Vector, Linear Algebra and Differential Equations.

Time Allowed: 03:00 Hours Maximum Marks: 100 Pass Marks: 33%

Note: Attempt Six questions in all, selecting two questions from section I & IV each and

SECTION – I

Q1:a). If CP and CD are semi-conjugate diameters of an ellipse with centre at C, show that eccentric angles of P and D differ by a right angle. **(9, 8)**

b). Show that an equation of the rectangular hyperbola $x^2 - y^2 = 1$ referred to its asymptotes as axes is $x'y' = -1/2$.

Q2: a). Find measure of angle of intersection of the curves $r = \cos 2\theta$ and $r = \sin\theta$ at $\left(\frac{1}{2}, \frac{\pi}{6}\right)$. **(9, 8)**

b). Show that pedal equation of the asteroid $x = a\cos^3\theta,\ y = a\sin^3\theta$ is $r^2 = a^2 - 3p^2$.

Q3: a). If $\underline{a} = t\,i - 3j + 2t\,k,\ \underline{b} = i - 2j + 2k,\ \underline{c} = 3i + t\,j - k$, then evaluate **(9, 8)**

$$\int_1^2 \underline{a}.\underline{b}\times\underline{c}\,dt$$

b). Establish the identity $\underline{a} = \frac{1}{2}\left[i\times(\underline{a}\times i) + j\times(\underline{a}\times j) + k\times(\underline{a}\times k)\right]$

Q4: a). Show that $\text{div}\frac{\underline{r}}{r^3} = 0$. **(9, 8)**

b). Find the scalar function φ when

$$\nabla\varphi = (y^2 - 2xyz^3)i + (3 + 2xy - x^2z^3)j + (6z^3 - 3x^2yz^2)k$$

SECTION – II

Q5: a). Find an equation of a straight line through the point $A(11, 4, -6)$ and intersecting at right angles to the straight line $\frac{x}{-1} = \frac{y-1}{1} = \frac{z}{-2}$ **(9, 8)**

b). Find an equation of the plane passing through the point $(2, -3, 1)$ and containing the line $x - 3 = 2y = 3z - 1$.

Q6: a). Find an equation of the sphere passing through the points $(0, -2, -4)$ and $(2, -1, -1)$ and having its centre on the line $2x - 3y = 0 = 5y + 2z$. **(9, 8)**

b). Find spherical coordinates of the point whose rectangular coordinates are $(-\sqrt{3}, 1, -2)$.

P.T.O

One question from II & III.

Q7: a). Show that $\sec^{-1} z = \frac{1}{i} \log\left(\frac{1+\sqrt{1-z^2}}{z}\right)$. **(9, 8)** t

b). If $\sin(\theta + i\varphi) = \cos\alpha + i\sin\alpha$, prove that $\cos^2\theta = \pm\sin\alpha$.

Q8: a). Prove that $64(\cos^8\theta + \sin^8\theta) = \cos 8\theta + 24\cos 4\theta + 35$. **(9, 8)**

b). Find direction of Qibla at Quetta

Latitude $\varphi = 30^\circ\, 15'$ N Longitude $\lambda = 67^\circ\, 0'$ E.

SECTION – III

Q9: a). If A is a matrix over R and $AA' = O$, show that $A = O$. **(8, 8)**

b). Solve the system of equations by Gauss- Jordon method.

$5x - 2y + z = 2,\ 3x + 2y + 7z = 3,\ x + y + 3z = 2.$

Q10: a). Prove that **(8, 8)**

$$\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^2$$

b). Express the vector (2, - 5, 3) in R^3 as a linear combination of the vectors (1, –3, 2), (2, – 4, –1) and (1, – 5, 7).

SECTION – IV

Q:11 a). Solve the initial value problem $\frac{dy}{dx} + \frac{y}{2x} = \frac{x}{y^3}$ $y(1) = 2$ **(8, 8)**

b). Solve the differential equation $y\,dy + x\,dx = \sqrt{x^2 + y^2}\,dx$

Q12: a). Solve $2y'' + y' + y = x^2 + 3\sin x$. **(8, 8)**

b). Solve $x^2 y'' - 3xy' + 5y = x^2 \sin(\ln x)$.