Model Paper

GOVERNMENT COLLEGE UNIVERSITY, FAISALABAD

BA/BSc (Part-I) Annual 2017

Subject: Mathematics (A-Course) Paper: (I) Course Code: MTH-301

Course Title: Calculus and Analytic Geometry

Time Allowed: 03:00 Hours Maximum Marks: 100 Pass Marks: 33%

Note: Attempt Six questions in all, selecting two question from section I and II each and one queston from III and IV.

Section-I

Question # 1: (a) Show that $\lim_{x\to 0}\frac{\cos x-1}{x}=0$. **(9,8)**

(b) If $P_n(x)=a_0x^n+a_1x^{n-1}+\ldots+a_{n-1}x+a_n$, then prove that $\lim_{x\to a}P_n(x)=P_n(a)$.

Question # 2: (a) Let v be the velocity of a particle at any given time t. Deduce that the acceleration of the particle at this instant is $\frac{dv}{dt}$. **(9,8)**

(b) Find the root of the equation $4\sin x=e^x$ in the interval $(0,0.5)$.

Question # 3: (a) Show that $\frac{d^n}{dx^n}\left(\frac{\ln x}{x}\right)=\frac{(-1)^n n!}{x^{n+1}}[\ln x-1-\frac{1}{2}-\frac{1}{3}-\ldots-\frac{1}{n}]$.

(9,8)

(b) Use L'Hospital rule to prove that $\lim_{x\to\infty}\left[\frac{a^{\frac{1}{x}}+b^{\frac{1}{x}}}{2}\right]^{x}=\sqrt{ab}, a>0, b>0$.

Question # 4: (a) Apply Taylor's theorem to prove that

$$(a+b)^m=a^m+\frac{m}{1!}a^{m-1}b+\frac{m(m-1)}{2!}a^{m-2}b^2+\ldots \text{ for all real } m, a>0, -a<b<a.$$

(b) Evaluate the given limits: $\lim_{x\to 0}\frac{\tanh x-\sinh x}{x^2}$. **(9,8)**

Section-II

Question # 5: (a) Find equations of tangent and normal to each of the following curves at the indicated point: $c^2(x^2+y^2)=x^2y^2$ at $\left(\frac{c}{\cos\theta},\frac{c}{\sin\theta}\right)$.

(b) Find the pedal equation of $r^m=a^m\cos m\theta$.

(9,8)

Question # 6: (a) Show that $\tan\psi=\frac{x\frac{dy}{dx}-y}{y\frac{dy}{dx}+x}$. **(9,8)**

(b) Sketch the graph of the given curves $r^2=a\sin 2\theta$.

Question # 7: (a) Show that the shortest between the lines $x+a=2y=-12z$ and $x=y+2a=6(z-a)$ is $2a$. **(9,8)**

(b) Express the given equation in cylindrical coordinates: $(x+y)^2-z^2+4=0$.

(P-T-O)

Question # 8: (a) Find an equation of the cone whose directrix is

Directrix: $y^2=x$, $z=4$ and whose vertex is at A (0,2,0)

(b) Find an equation of the tangent plane to the sphere $x^2+y^2+z^2-4x+2y-6z=0$ at the point P(3,2,5) (9, 8)

Section III

Question # 9: (a) Find the asymptotes of $2xy+2y=(x-2)^2$ (8, 8)

(b) Show that the height of an open cylinder of given surface S and greatest Volume is equal to the radius of the base.

Question # 10: (a) Find the equations of the tangents at the multiple points of the curve

$(y-z)^2 = x(x-1)^2$ (8, 8)

(b) Find the intrinsic equation of $\gamma = a(1-\cos\theta)$

Section IV

Question # 11: (a) Evaluate $\int x^5 e^{x^3} dx$. (8, 8)

(b) Integrate $1/(x^4+1)$ with respect to x.

Question # 12: (a) Calculate $\int_0^{\pi/2} \ln \sin x \, dx$. (8, 8)

(b) Use Trapezoidal Rule to approximate the integral $\int_0^{\pi} \sin x \, dx$ with n=6.

Model Paper

GOVERNMENT COLLEGE UNIVERSITY, FAISALABAD

B.A/B.Sc (Part-II) Annual 2017

Subject: Mathematics (A-Course) Paper: (II) Course Code: MTH-401

Course Title: Linear Algebra and Differntial Equations

Time Allowed: 03:00 Hours Maximum Marks: 100 Pass Marks: 33%

Note: Attempt Six questions in all, selecting two questions from section I & IV each and one Question from II & III.

SECTION-I

Q.1 (a) Prove that the product of matrices. **(9,8)**

$$A=\begin{bmatrix}\cos^2\alpha & \cos\alpha\sin\alpha\\ \cos\alpha\sin\alpha & \sin^2\alpha\end{bmatrix} \quad \text{and} \quad B=\begin{bmatrix}\cos^2\beta & \cos\beta\sin\beta\\ \cos\beta\sin\beta & \sin^2\beta\end{bmatrix}$$

is the zero matrix if α and β differ by $\frac{(2n+1)\pi}{2}$, where n is an integer.

(b) Prove that $\begin{vmatrix} a^3 & 3a^2 & 3a & 1\\ a^2 & a^2+2a & 2a+1 & 1\\ a & 2a+1 & a+2 & 1\\ 1 & 3 & 3 & 1\end{vmatrix}=(a-1)^6$.

Q.2 (a) A yarn merchant sells brands A, B, C of yarn each of which is a blend of Pakistani, Egyptian and American cotton in ratio 1:2:1, 2:1:1 and 2:0:2, respectively. If one kilogram of A, B, C costs 40,50 and 60 rupees respectively, find the cost of a kilogram of cotton of each country. **(9,8)**

(b) Let A and B be nonsingular matrices of the same order then AB is a nonsingular. Then prove that $(AB)^{-1}=B^{-1}A^{-1}$.

Q.3 (a) Show that the vectors $\{(1,2,3),(0,1,2),(0,0,1)\}$ generate R^3. **(9,8)**

(b) Let U and W be subspaces of a vector space V over a field F. Then prove that $U\cap W$ is also a subspace of V.

Q.4 (a) A linear transformation $T:U\to V$ is one-one if and only if N(T)={0}. **(9,8)**

(b) Find a basis and dimension of R(T) where $T:R^3\to R^3$ is defined by

$$T(x_1,x_2,x_3)=(x_1+2x_2-x_3,x_2+x_3,x_1+x_2-2x_3).$$

SECTION-II

Q.5 (a) Let $u=(u_1,u_2)$ and $v=(v_1,v_2)$ belong to R^2 verify that

$\langle u,v\rangle=u_1v_1-2u_1v_2-2u_2v_1+5u_2v_2$ is an inner product on R^2. **(8,8)**

(b) Show that the rows (columns) of the matrix $\begin{bmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}$ form an orthonormal set.

(P-T-O)

Q.6 (a) Find the eigen values and corresponding eigen vectors of the matrix $\begin{bmatrix} 1 & 2 & 2 \\ 1 & 2 & -1 \\ -1 & 1 & 4 \end{bmatrix}$. **(8,8)**

(b) If λ is an eigen value of an orthonrmal matrix, then $|\lambda| = 1$.

SECTION-III

Q.7 (a) Solve $x\dfrac{dy}{dx} + y = y^2 \ln x$. **(8,8)**

(b) Solve $p = \tan\left(x - \dfrac{p}{1+p^2}\right)$.

Q.8 (a) Solve $(xy^2 + 2x^2y^3)dx + (x^2y - x^3y^2)dy = 0$. **(8,8)**

(b) Find an equation of orthonormal trajectory of the curve $r^n \cos n\theta = a^n$.

SECTION-IV

Q.9 (a) Solve $(D^3 - D^2 + D - 1)y = 4\sin x$. **(9,8)**

(b) Solve $x^4 \dfrac{d^3y}{dx^3} + 2x^3 \dfrac{d^2y}{dx^2} - x^2 \dfrac{dy}{dx} + xy = 1$.

Q.10 (a) Solve particular solution: $\dfrac{d^2y}{dx^2} - 3\dfrac{dy}{dx} + 2y = (1+e^{-x})^{-1}$. **(9,8)**

(b) Solve: $\dfrac{d^2y}{dx^2} + y = \sec^3 x$.

Q.11 (a) Compute (i) $L(\sin at)$ (ii) $L^{-1}\left(\dfrac{3s+1}{s^2 - 6s + 18}\right)$. **(9,8)**

(b) Apply the power series method to solve: $y' - ky = 0$.

Q.12 Use Laplace transformation method to solve: **(9,8)**

(a) $\dfrac{d^2y}{dt^2} - 2\dfrac{dy}{dt} + y = te^t;\ \ y(0) = 0 = y'(0)$.

(b) $\dfrac{dy}{dt} + 4y = 2e^t - 4e^{-t};\ \ y(0) = 0$.